Preprint
UCRL-JC-142236

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This article was submitted to
19th International Symposium on Ballistics 2001
Interlaken, Switzerland
May 7-11, 2001

January 25, 2001

U.S. Department of Energy

Lawrence
Livermore
National
Laboratory

COUPLED MAP LATTICE MODEL OF JET BREAKUP

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An alternative approach is described to evaluate the statistical nature of the breakup of shaped charge liners. Experimental data from ductile and brittle copper jets are analyzed in terms of velocity gradient, deviation of ΔV from linearity, R/S analysis, and the Hurst exponent within the coupled map lattice model. One-dimensional simulations containing 600 zones of equal mass and using distinctly different force-displacement curves are generated to simulate ductile and brittle behavior. A particle separates from the stretching jet when an element of material reaches the failure criterion. A simple model of a stretching rod using brittle, semi-brittle, and ductile force-displacement curves is in agreement with the experimental results for the Hurst exponent and the phase portraits and indicates that breakup is a correlated phenomenon.

INTRODUCTION

Numerous models have been proposed in the literature to describe the particulation of shaped charge jets. Hirsch [1] developed a breakup time model related to the initial jet radius and the particle velocity difference. Chou and Carleone [2] suggested that plastic instability controlled by the material strength dictates the breakup process. Similarly, Walsh [3] evaluated the plastic instability and the break-up process. Chanteret [4,5] evaluated the velocity gradients and shaped charge jet length as well as the influence of material density on shaped charge performance. Zernow and Chapyak [6] developed a 3D computation model for breakup that considers the double-helix surface perturbations. Mayseless et al. [7] suggested a novel approach to characterize breakup time based on the observation that the breakup distance for a given liner material and geometry was a constant.

In spite of the large experimental data bank of shaped charge jet particulation, very little attention has been applied to the statistical analysis of the breakup. It has been suggested that appropriate application of various statistical methods will lead to a better understanding of the influence of shaped charge design and shaped charge

*This work was performed under the auspices of the U.S. Department of Energy by University of California Lawrence Livermore National Laboratory under contract No. W7405-Eng-48.

material properties. Schwartz et al. [9] evaluated the statistical nature of the breakup phenomenon in silver jets using the return map of Curry and York. In that investigation, two high purity silver liners were fabricated in the standard 81-mm shaped charge design. The methods used in characterizing nonlinear dynamical systems were applied to the measured velocity fluctuations for two silver shaped charge jets. Evidence was presented that the fluctuations exhibit a non-Gaussian behavior indicative of an underlying nonlinear dynamical system. The use of phase portraits gave visual clues that the origin of the fluctuations may be due to a quasiperiodic route to chaos involving two nonlinearly coupled resonators analogous to Rayleigh-Benard Convection.

In the present work, the role of nonlinear fluctuations resulting in the statistical particulation of shaped charge jets is investigated for two copper jets using a coupled map lattice (CML) model. Such models have been successful for studying instabilities in open fluid flows. The nonlinear material dependence of stress on the strain may be coupled to the inertial degrees of freedom by means of CML. The resulting nonlinear stress and velocity fluctuations may be studied using the techniques of nonlinear dynamics. Of particular interest is to study the differences in the phase portraits for brittle versus ductile materials and for different stochastic models for impurities and the resulting influence on the constitutive behavior. It is expected that brittle materials will manifest a well-defined attractor in a phase portrait. The tendency for more ductile materials to have a more diffuse attractor is investigated for different stress-strain relations and impurity levels. It is hoped that the ideas presented in this paper will allow useful information to be extracted from shaped charge jet data that will result in a better understanding of jet break-up.

EXPERIMENTAL DATA

The copper shaped-charge liners were produced from OFE 99.99% copper, Hitachi C10100 bar stock. The liners were back extruded using a standard cold-forge process into the shape of hollow cones (base inner diameter = 81 mm, apex angle = 42^{o}). After forging, the liners were annealed in order to stabilize the microstructure for subsequent sulfur doping as described in [9]. The grain size of the liners was measured using standard metallographic techniques, and the breakup times were determined from the flash x-ray radiographs of the jets. The experiments use a high precision, 81mm Cu shaped charge design, cast loaded with Octol high explosive. The two liners vary in sulfur content and grain size to produce breakup times of 193 and 147 μsec for the ductile and brittle jets, respectively. Long standoff flash x-rays are used in order to characterize the particulated shaped charge jets as shown in Fig 1. The velocity versus particle number for two different shaped charges is shown in Fig 2a and b. The deviations of the velocity from a fitted straight line versus particle number are shown in Fig 2c and d. Phase portrait maps for the two jets are presented in Fig 2 e and f.

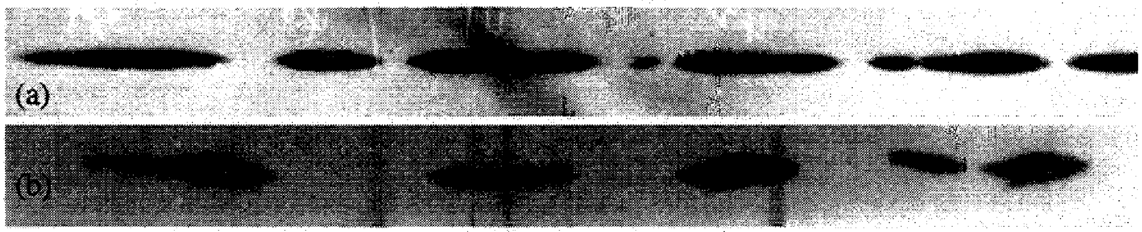


FIGURE 1. Radiographs for the two jets (a) ductile behavior, and (b) brittle behavior.

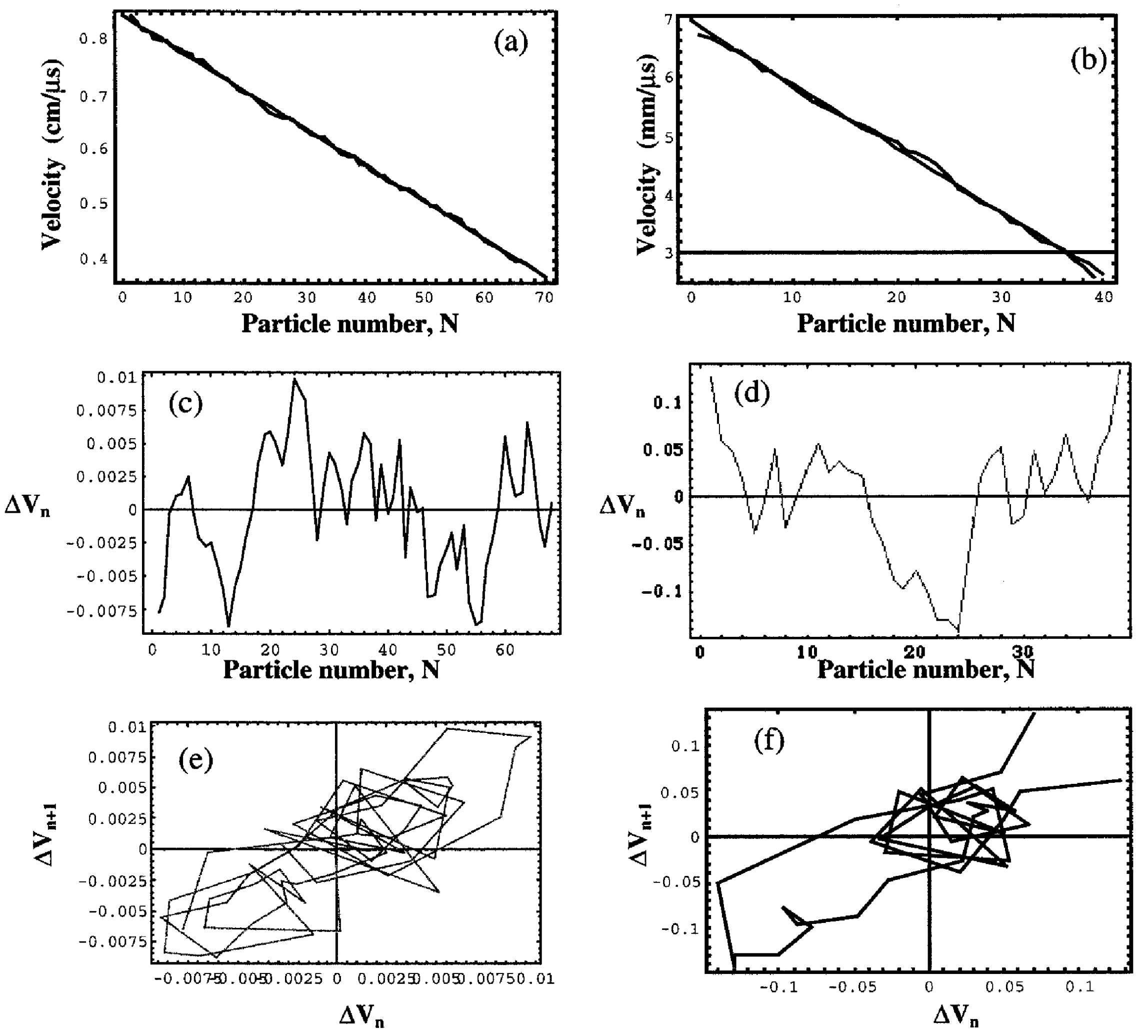


FIGURE 2. (a) Velocity versus particle number for the ductile jet, (b) velocity versus particle number for the brittle jet, (c) velocity deviation from a straight line for the ductile jet, (d) velocity deviation from a straight line for the brittle jet, (e) diffuse phase portrait of the ductile jet, and (f) strong phase portrait of the brittle jet.

STATISTICAL COUPLED MAP LATTICE MODEL

The complex nonlinear dynamics of spatially extended systems can exhibit a rich variety of structures and patterns across many time and space scales. Coupled Map Lattice (CML) models [10] have been successful in studying spatiotemporal pattern formation and the role that deterministic fluctuations play in the generation of observed macroscopic states. For example, the generation of instabilities and the transition to turbulence in hydrodynamic open flows has been modeled using CML. The particulation of a shaped charge jet is statistical in character as can be seen, for example, by measuring jet particle length, interparticle distance and velocity, or the deviation in particle velocity from the velocity determined from the average velocity gradient, Γ. It is expected that the fluctuations will be highly correlated if they reflect an underlying dynamical attractor in phase space. If the dimension of the attractor is large or there exists more than one low dimensional attractor, it is expected that the correlations may be substantially weakened. Material heterogeneities are also expected to alter the spectrum of fluctuations. In the present study, two different methods are used to characterize correlations of the velocity fluctuations for both the CML model calculations and in the experimental data of two Cu shaped charge jets that differ significantly in the time to breakup time, (147μs and 193μs). The first method is known as R/S analysis and is used to extract a correlation exponent, known as the Hurst exponent. The second method involves the construction of phase portraits and provides a graphical representation of trajectories in phase space that can reveal the existence of an underlying attractor. The CML is a one-dimensional chain of N-1 linked nonlinear functions, f, which depends only on the spatial coordinate, x. This function represents a force-distance, or equivalently, a stress-strain relation that characterizes the strength of the material. The force acts on adjacent zones of equal mass, m, and serves to alter the kinetic energy of each zone according to:

$$m_j \frac{dv_j}{dt} = f(x_{j+1} - x_j - \Delta_0) - f(x_j - x_{j-1} - \Delta_0) \ . \tag{1}$$

The function, f, is taken to have the form,

$$f(x) = f_{\max} \frac{2\beta x}{1+(\beta x)^2}, \tag{2}$$

and is approximately linear for small x, corresponding to an elastic regime close to the unstressed length, Δ_0. The force takes its maximum value of f_{max} at a net displacement of β^{-1}.

In order to simulate the breaking of two adjacent spatial zones, the force is set to zero according to a probabilistic rule that depends also on the separation of the zones,

$$p(x) = e^{-k(1-\frac{x}{x_{\max}})^2} \quad . \tag{3}$$

The initial state of the jet at t = 0 sec consists of N zones of equal mass, and zone j has a width given by

$$\Delta x_j = \Delta x_0 e^{\Gamma \delta \tau (j-1)}, \tag{4}$$

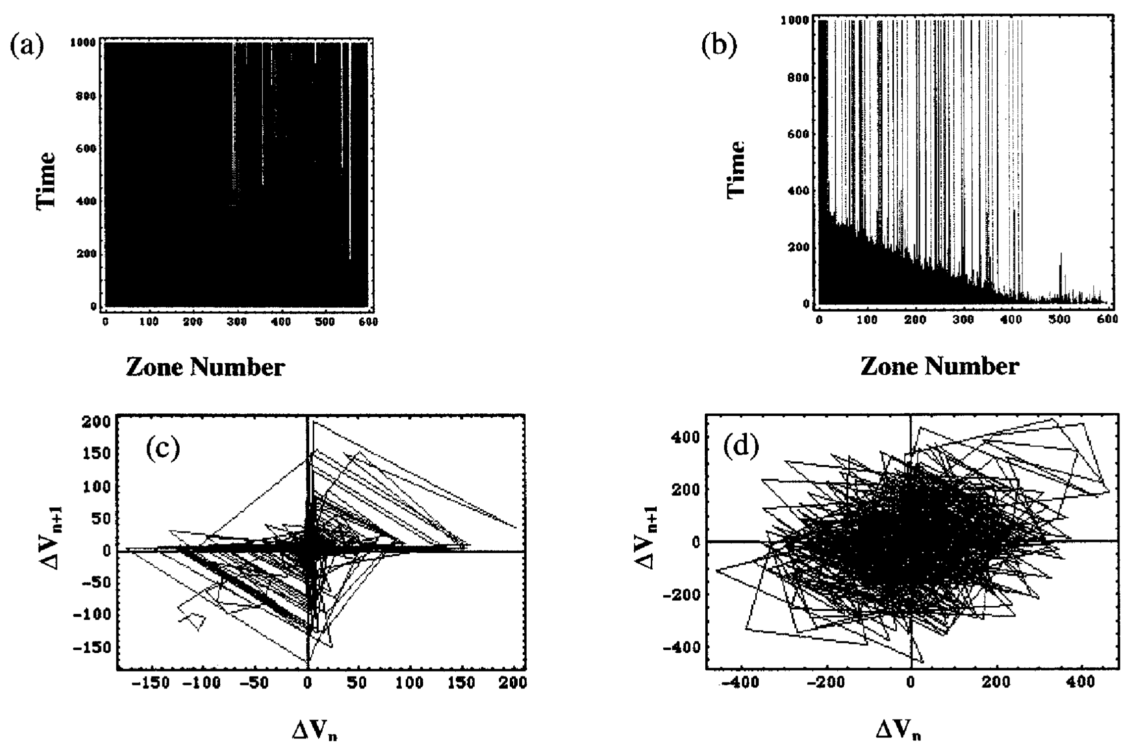
for a strain rate, Γ. The parameters are chosen such that the maximum strain in the last zone of the jet is less than the fracture strain for the particular material strength model in view. Presented in Table 1 are all of the model parameters used for two different brittle cases and one ductile case. The fracture strain for the ductile case is chosen to be twice that of the brittle case which is consistent with the 46 μs difference in breakup times (i.e., 193 μs – 147 μs) for the two experimental Cu jets.

Table 1. Parameters for the CML Model

Strength Model	N	Δx_0 (cm)	Δt (s)	k	Γ (s^{-1})	P_0	ε_f
Brittle	600	0.10	$3.0\ 10^{-7}$	100.	10^4	10^{-2}	1.5
Semi-Brittle	600	0.10	$3.0\ 10^{-7}$	10.	10^4	$5.0\ 10^{-3}$	1.5
Ductile	600	0.10	$3.0\ 10^{-7}$	10.	10^4	$5.0\ 10^{-3}$	3.0

Initially, the velocity and position of each node is specified. The relative positions between the center of mass of each adjacent fixed mass zone are then used to compute the force on a given mass. Setting the value of the force to zero simulates the separation of two adjacent masses. A random number generator and the probabilistic rule determine the criterion for separation of adjacent zones. Separations occur first near the jet tip (high velocity) due to the higher strains at t = 0. The probability that they occur away from the tip increases with time. Fig 3 is a density plot showing the position and time that a separation occurs in the calculation for two different strength models. The brittle case (Fig 3a) exhibits a more deterministic time ordering as the position of the separations move from the jet tip towards the tail. The ductile case (Fig 3b), on the other hand, is much less deterministic in its time ordering. The phase portraits of the velocity fluctuations, ΔV, for both the brittle case (Fig 3c) and ductile case (Fig 3c) show that the brittle case is much less diffuse in phase space than the ductile case. This is qualitatively similar to the phase portraits shown for Cu jets in Fig. 2 e and f. The phase portrait for the semi-brittle case also showed the same trends, being more diffuse than the brittle case but less diffuse than the ductile case.

The phase portraits provide a graphical representation of velocity fluctuation correlations along the jet. A more quantitative method, known as R/S analysis allows one to extract the Hurst exponent (H), named after the developer of this method [11]. The Hurst exponent can take on values between 0 and 1. An H value of 1/2 suggests no correlation between a particular value and the next, whereas, H >1/2, suggest a positive correlation and H<1/2, suggests two values are anti-correlated. The value of H for the silver jets reported in [9], have an H = 0.7. The value of H for the Cu jets are both H ~ 0.7. The R/S plot for the CML computations is shown in Fig 3e for the brittle case and has a value of H = 0.85 consistent with a highly deterministic breakup. The R/S plot for the ductile case is shown in Fig 3f and tends to lie between two slopes corresponding to H = 0.7 and 0.8. The same jet can show regions with different values of H.



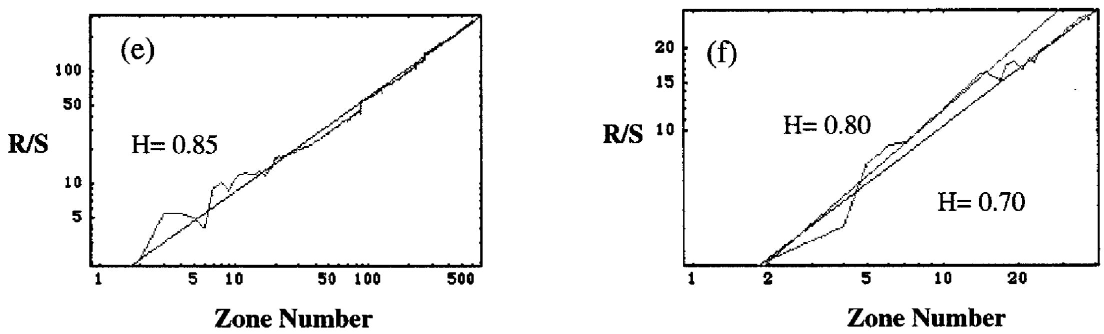


FIGURE 3. (a) Density plot of breaking times for each zone for a brittle jet, and (b) density plot of breaking times for each zone for ductile jet. Units of time are 0.3 μs. (c) strong phase portrait of the brittle jet, (d) diffuse phase portrait of the ductile jet, (e) R/S plot of velocity fluctuations for brittle jet, and (f) R/S plot for the ductile case. The slopes of the straight lines on the double-logarithmic plots (i.e., Hurst exponents) are shown. A value of $H = 1$, implies perfect correlation and a value of 1/2 is random, showing no correlation. A value of 0.7 is typical in the experimental jets studied to date.

SUMMARY

We are exploring alternative methods to evaluate the statistical nature of particulation of shaped charge jets. As a test case, we experimentally determined the breakup time of two copper jets, one exhibiting brittle behavior, and the other ductile behavior. The deviation of linearity in particle velocities is used to determine the phase portrait. Two distinctly different force-displacement curves are generated to simulate ductile and brittle behavior. A simple model of a stretching rod containing 600 zones of equal mass starts the calculation. When an element of material reaches the failure criterion, a particle separates from the stretching jet. A comparison of the experimental and simulated results shows reasonable agreement and suggests this approach is worthy of further study.

ACKNOWLEDGMENTS

The authors acknowledge the support of the Joint DoD/DOE Munitions Technology Development Program. This work was performed under the auspices of the U.S. Department of Energy by the University of California, Lawrence Livermore National Laboratory under Contract No. W-7405-Eng-48.

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